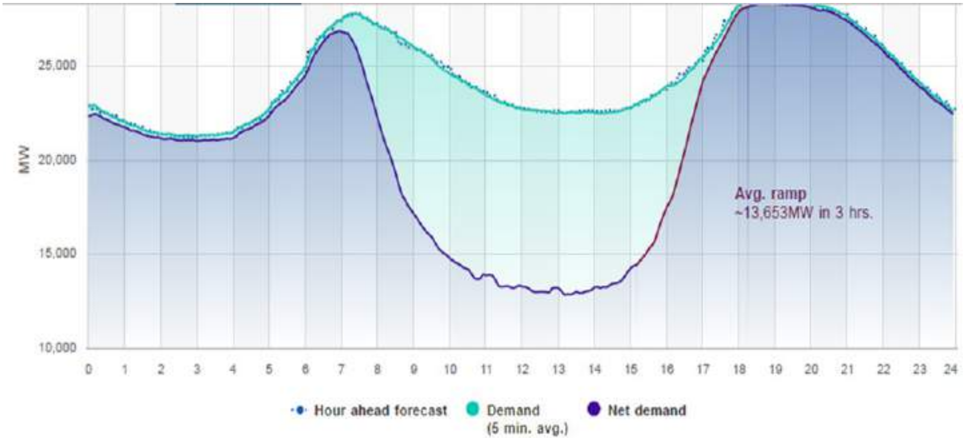
Storing Power: Market Structure Matters

David Andrés-Cerezo and Natalia Fabra

March 6, 2020

The "Duck curve"



The Duck: CAISO Total Demand and Net (of Solar and Wind) Demand for Feb 7, 2019

(source: http://www.caiso.com/TodaysOutlook/Pages/default.aspx)

Introduction

- **Storage's key role in the energy transition:**
  - Make a more efficient use of existing resources (e.g. excess renewables).
  - Provide energy when renewables are not available.
  - Reduce the need to invest in (polluting) back-up generation capacity.

Introduction

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  - Make a more efficient use of existing resources (e.g. excess renewables).
  - Provide energy when renewables are not available.
  - Reduce the need to invest in (polluting) back-up generation capacity.
- **Goals of the paper:**
  1. Analyze whether the private and social incentives for **investing** in storage are aligned.
  2. Understand decentralized **storage operation** and its impact on market outcomes.
  3. Asses how this depends on **market structure**.

Storing technologies and market structure

- Different types of storage facilities...

Figure: Pumped hydro

Figure: Grid-scale batteries

Figure: Electric vehicles

Storing technologies and market structure

- Different types of storage facilities...

Figure: Pumped hydro

Figure: Grid-scale batteries

Figure: Electric vehicles

- ...imply different **horizontal and vertical market structures:**
  - Horizontal: Storage can be competitive vs. strategic
  - Horizontal: Generation can be competitive vs. strategic
  - Vertical: stand-alone vs. vertically integrated storage

Example: Vertically Integrated Storage Firms

FEBRUARY 27

Tesla's massive 1GWh Megapack battery project with PG&E is approved

Fred Lambert - Feb. 27th 2020 9:04 am ET

98 Comments | Facebook | Twitter | Pinterest | LinkedIn | Reddit

Tesla's massive project to deploy 1GWh of Megapacks to create a giant energy storage system in California with PG&E has received approval from the local authorities.

We first learned of the project at PG&E's Moss Landing substation when they submitted it to CPUC and the company was in talks with Tesla in 2017.

It involves four separate energy storage projects and two of them should become the world's largest battery systems.

Example: Vertically Integrated Storage Firms

Tâmega: one of the largest hydroelectric projects developed in Europe in the last 25 years

Iberdrola group is investing more than 1.5 billion euros in constructing the Tâmega hydroelectric complex in northern Portugal, which will involve building three dams and three power plants (Gouvães, Daivões and Alto Tâmega) with a combined capacity of 1,158 MW.

The Tâmega project involves the construction of three new power plants: Gouvães, Daivões and Alto Tâmega, which will be erected over the Tâmega River, a tributary of the Duero in the north of Portugal, close to Oporto. The three power plants will have an installed capacity totaling 1,158 MW, representing an increase of 6% in the total installed electrical power in the country.

The complex will be capable of producing 1,766 GWh per year, enough to meet the energy needs of the neighbouring towns and the cities of Braga and Guimarães (440,000 homes). Furthermore, this large renewable infrastructure will have sufficient storage capacity to serve two million Portuguese households for an entire day.

Regulatory debate

- **Ownership structure** $\rightarrow$ Hotly debated area:
  - California $\rightarrow$ Utilities mandated to invest in storage capacity.
  - Texas $\rightarrow$ Utilities not permitted to own storage capacity.
  - FERC $\rightarrow$ System Operators not allowed to use storage.
- **Investment decisions** $\rightarrow$ Recent orders by FERC and European Commission assume that the market will provide optimal incentives for investment.
  - Do not contemplate the establishment of new markets and new policies.
  - Several EU countries have started to run auctions for energy storage.

Example: Auctions for Energy Storage

Ireland allocates 110 MW in large scale storage auction

Originally intended to commission 140 MW of storage, the tender drew three winning projects: a 50 MW system and two 30 MW facilities. Eirgrid has estimated the total value of the contracts at around €6 million per year.

OCTOBER 4, 2019 EMILIANO BELLINI

ENERGY MANAGEMENT SYSTEMS | ENERGY STORAGE | GRIDS & INTEGRATION | MARKETS | POLICY | UTILITY SCALE STORAGE | IRELAND

Goals and methodology

- **Relevant questions:**
  1. What will storage facilities be managed?
  2. Will investment in storage capacity be socially optimal?
  3. Do storage facilities confer market power? What is the effect on prices and efficiency?
  4. How do the above questions depend on who owns the storage?

Goals and methodology

- **Relevant questions:**
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  4. How do the above questions depend on who owns the storage?

- **Methodology:**
  1. Theory $\rightarrow$ Stylized model to understand main forces at work
  2. Empirical analysis $\rightarrow$ Simulation for Spanish electricity market

Related literature

- **Related literature:**
  - **Storage and commodity speculation.**
    - McLaren (1999) Williams&Wright (1991); Mitraille, Thille (2014); etc...
  - **Natural resource extraction.**
    - Hotelling (1931); Salant (1976); etc...
  - **Electricity storage.**
    - Schmalensee (2019); Ambec&Crampes (2018); Karaduman (2020); Crampes&Trochet (2019)...

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  - **Electricity storage.**
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- **Contributions of this paper:**
  - General framework to compare different market structures.
  - Joint analysis of production and investment decisions.

Outline

Modelling set-up

- **Demand**
  - Price- inelastic demand θ; consumers' valuation v.
  - θ is distributed according to a **symmetric** $G(\theta)$ in $\left[\underline{\theta}, \overline{\theta}\right]$.
    - θ can be interpreted as demand net of renewables.
    - Known at the production stage $\rightarrow$ **focus on seasonal variation.**
    - **Stationarity:** $G(\theta)$ captures the frequency of each demand level.

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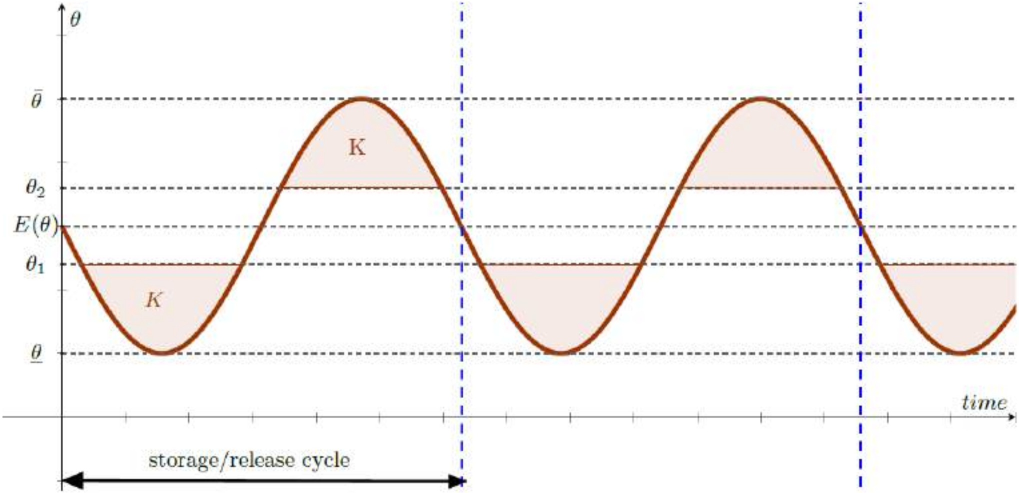
- **Generation**
  - Generation costs $c(Q)$: increasing and convex.
- **Storage**
  - Storage capacity K (in MWh); Investment costs $C(K)$: increasing and convex.
  - $q^B(\theta), q^S(\theta)$: quantities bought (B) and sold (S) by the storage facility.
- **Timing**
  1. Investments in storage capacity.
  2. Production and pricing decisions.

Demand pattern: an illustration



Outline

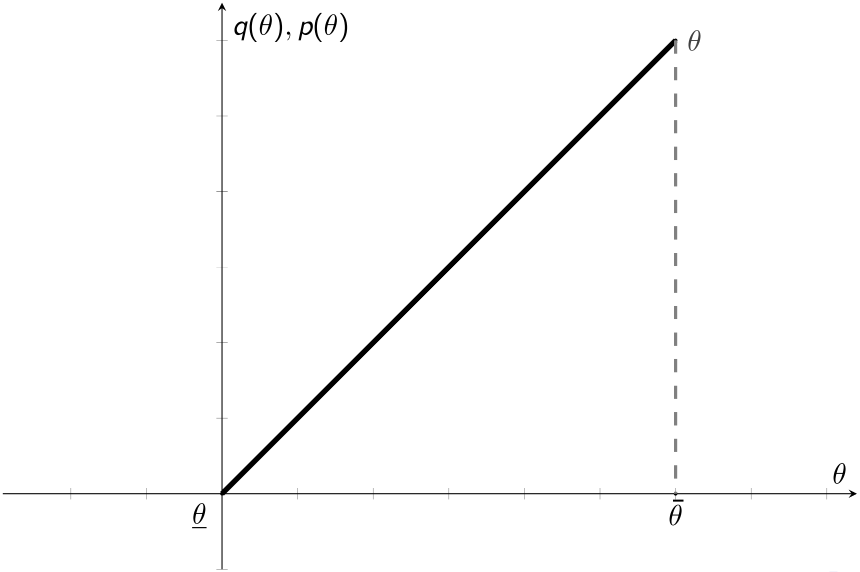
First Best Problem

Welfare is gross consumer surplus minus production and investment costs:

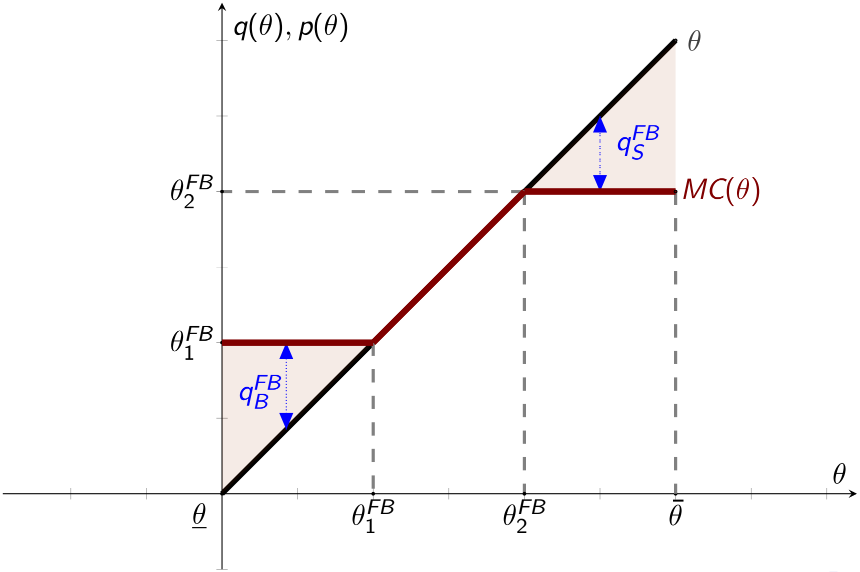
$$\max_{q^B(\theta),q^S(\theta),K} \mathcal{W} = \int_{\underline{\theta}}^{\overline{\theta}} v\theta dG(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} c\left(\theta - q^S(\theta) + q^B(\theta)\right) dG(\theta) - C(K)$$

$$\begin{aligned} \text{s.t. } (\lambda) \quad &: \quad \int_{\underline{\theta}}^{\overline{\theta}} q^B(\theta) dG(\theta) \geq \int_{\underline{\theta}}^{\overline{\theta}} q^S(\theta) dG(\theta) \\ (\mu) \quad &: \quad \int_{\underline{\theta}}^{\overline{\theta}} q^B(\theta) dG(\theta) \leq K \end{aligned}$$

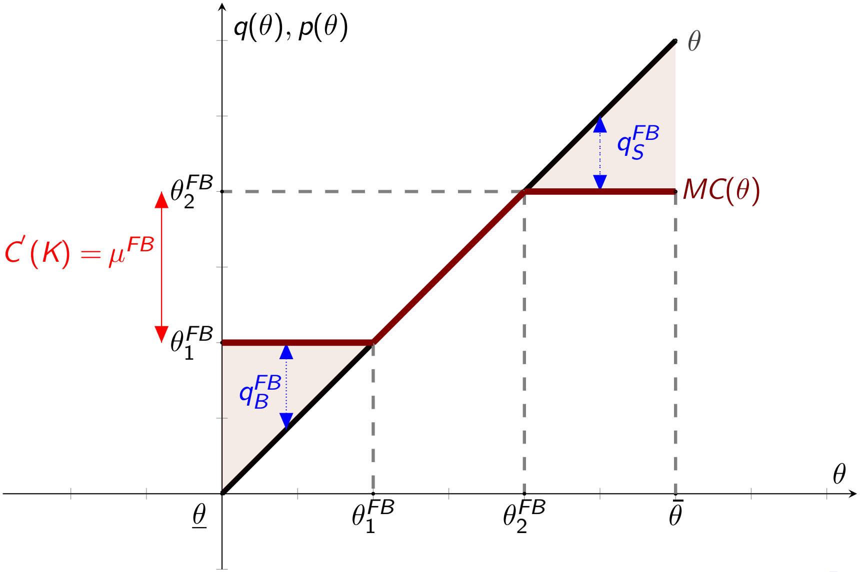
First Best



First Best


$q(\theta), p(\theta)$
θ
q_S^{FB}
θ_2^{FB}
$MC(\theta)$
θ_1^{FB}
q_B^{FB}
θ
$\underline{\theta}$
θ_1^{FB}
θ_2^{FB}
$\bar{\theta}$

First Best



$q(\theta), p(\theta)$
θ
q_S^{FB}
θ_2^{FB}
$MC(\theta)$
$C'(K) = \mu^{FB}$
θ_1^{FB}
q_B^{FB}
θ
$\underline{\theta}$
θ_1^{FB}
θ_2^{FB}
$\bar{\theta}$

First Best

- **Optimal storage management:**
  - Store when demand is low and release when demand is high.
  - Equalization of marginal costs within storing and releasing regions.
  - Minimization of total costs of production.

- **Optimal investment in storage:**
  - Marginal benefit $\Rightarrow$ Marginal cost saving from storing one more unit of output.
    - Store an extra unit that costs θ_1^{FB} to substitute a unit that costs θ_2^{FB}.
  - No full marginal cost equalization.
    - Otherwise, marginal value of capacity would be zero $>$ marginal capacity costs.

First Best

Outline

Horizontal market structure: Production

- Existing assets are owned by:
  - a **dominant firm**: $\alpha \in (0,1)$ share, with costs $c_D(q) = \frac{q^2}{2\alpha}$.
  - a **competitive fringe**: $(1-\alpha) \in (0,1)$ share, with costs $c_F(q) = \frac{q^2}{2(1-\alpha)}$.

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  - a **competitive fringe**: $(1-\alpha) \in (0,1)$ share, with costs $c_F(q) = \frac{q^2}{2(1-\alpha)}$.
- Fringe produces whenever price $>$ marginal cost $\rightarrow$ fringe's supply curve:

$$q_F = c_F'(q) = (1-\alpha)\, p(\theta)$$

- Equilibrium market price on the fringe's supply curve:

$$p(\theta, q^D, q^S, q^B) = \frac{\theta - q^D(\theta) - q^S(\theta) + q^B(\theta)}{1-\alpha}$$

No storage

- Dominant firm maximizes profits over the residual demand:

$$\max_{q^D(\theta)} \pi^D = p(\theta, q^D(\theta))\, q^D(\theta) - \frac{[q^D(\theta)]^2}{2\alpha}$$

- **Equilibrium outcome:**

$$q^D(\theta) = \theta \frac{\alpha}{1+\alpha}$$

- Constant mark-up equal to α.
- Distorted market shares:
  - Dominant produces less than at the efficient solution $\alpha/(1+\alpha) < \alpha$.
  - Fringe produces more than at the efficient solution $1/(1+\alpha) > 1-\alpha$

Outline

Second Best: production stage

- **Nash equilibrium** (open-loop strategies).
- **Dominant firm** chooses q_D to maximize "intertemporal" profits:

$$\max_{q^D(\theta)} \pi = \int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{\theta - q^D(\theta) - q^S(\theta) - q^B(\theta)}{1-\alpha} q^D(\theta) - \frac{\left[q^D(\theta)\right]^2}{2\alpha} \right) dG(\theta)$$

- **Social planner** chooses q_S and q_B to maximize total welfare:

$$\max_{q_B(\theta), q_S(\theta)} \mathcal{W} = \int_{\underline{\theta}}^{\overline{\theta}} v\theta dG(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{\left[q^D(\theta)\right]^2}{2\alpha} + \frac{\left[\theta - q^D(\theta) - q^S(\theta) - q^B(\theta)\right]^2}{2(1-\alpha)} \right) dG(\theta)$$

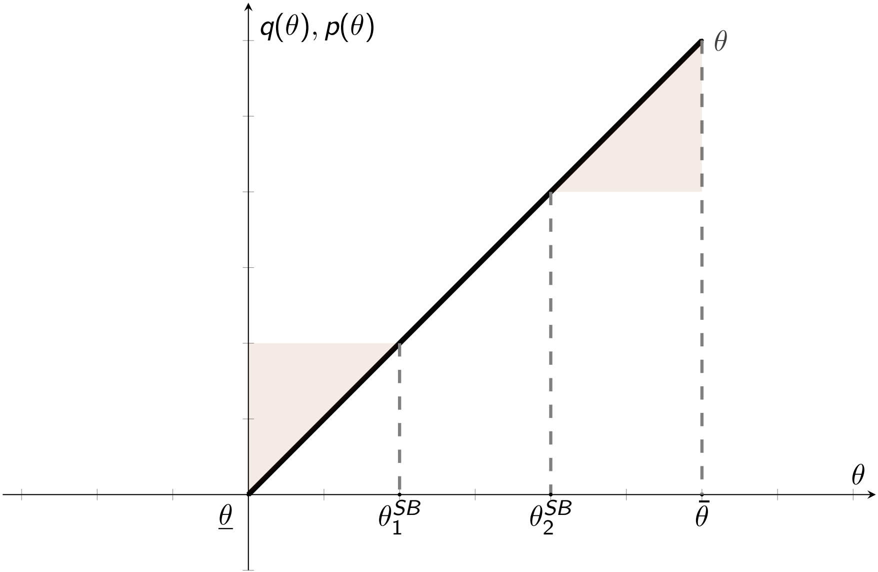
subject to the storage constraints.

Second Best: investment stage

- **Social planner**, given the optimal production decisions $q^D(\theta, K)$, $q^B(\theta, K)$ and $q^S(\theta, K)$, chooses K to maximize:

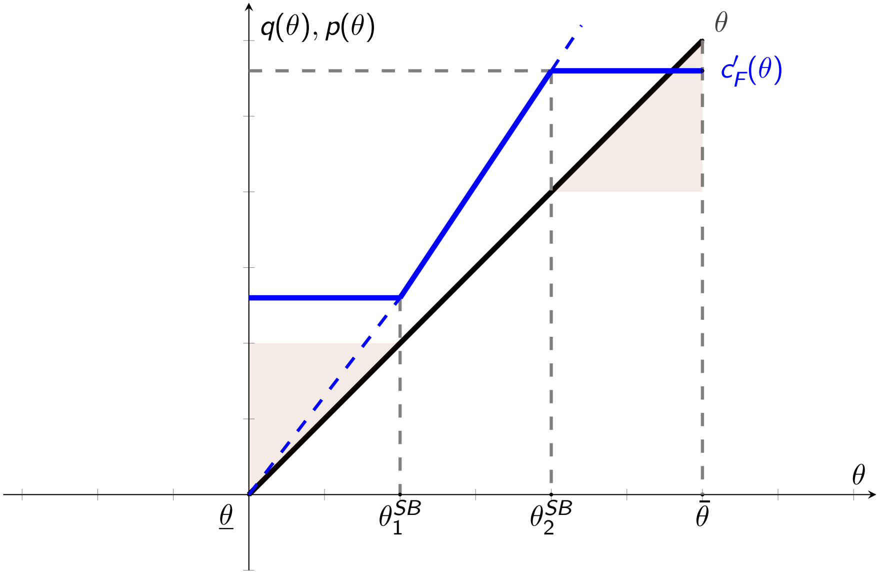
$$\max_K \left\{ \int_{\underline{\theta}}^{\overline{\theta}} v\theta dG(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{\left[q^D(\theta, K)\right]^2}{2\alpha} + \frac{\left[\theta - q^D(\theta, K) - q^S(\theta, K) - q^B(\theta, K)\right]^2}{2(1-\alpha)} \right) dG(\theta) - \right.$$

Second Best



$q(\theta), p(\theta)$
θ
θ
$\underline{\theta}$
θ_1^{SB}
θ_2^{SB}
$\overline{\theta}$

Second Best



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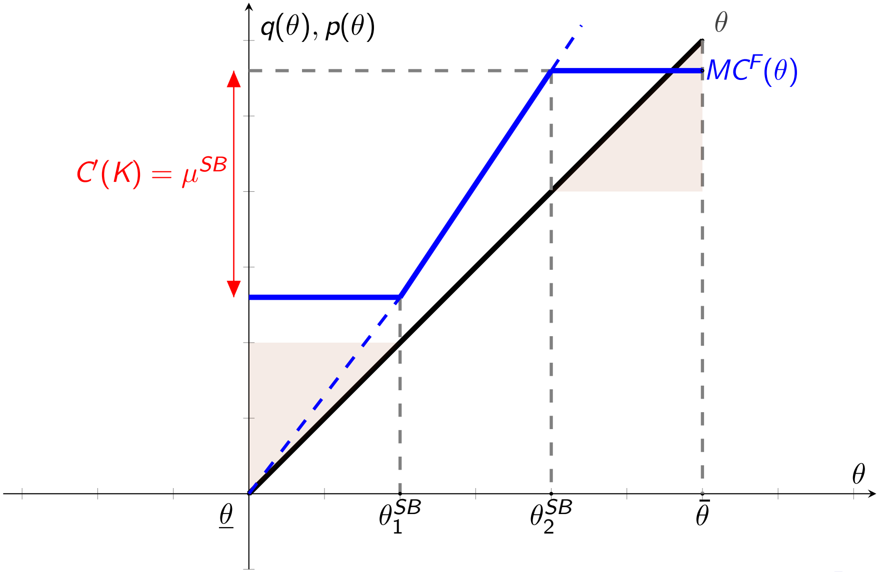

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θ_1^{SB}
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Second Best

- **Optimal storage management:**
  - Similar to FB.
  - Equalization of the **fringe's marginal costs** within storing and releasing regions (since q^D is fixed).
  - Minimization of marginal cost of production (given the dominant's behavior).

- **Optimal investment in storage:**
  - Marginal benefit $\Rightarrow$ Marginal cost saving from adding one unit of storage.
  - The difference in the marginal costs of the fringe is greater than the difference in marginal costs if production was fully efficient
    - Market power in production increases the marginal value of storage.

Outline

Model set-up

First Best

Market solution

- Second Best
- Competitive storage
- Independent storage monopolist
- Vertically integrated firm

Welfare comparison

Competitive storage

Perfect competition:

1. Large set of small owners (e.g. electric cars).
2. They take production prices as given.
3. Free entry in the storage market $\Rightarrow$ zero-profit condition.

- **Production stage:** Operate storage so as to max. arbitrage profits (taking prices as given)

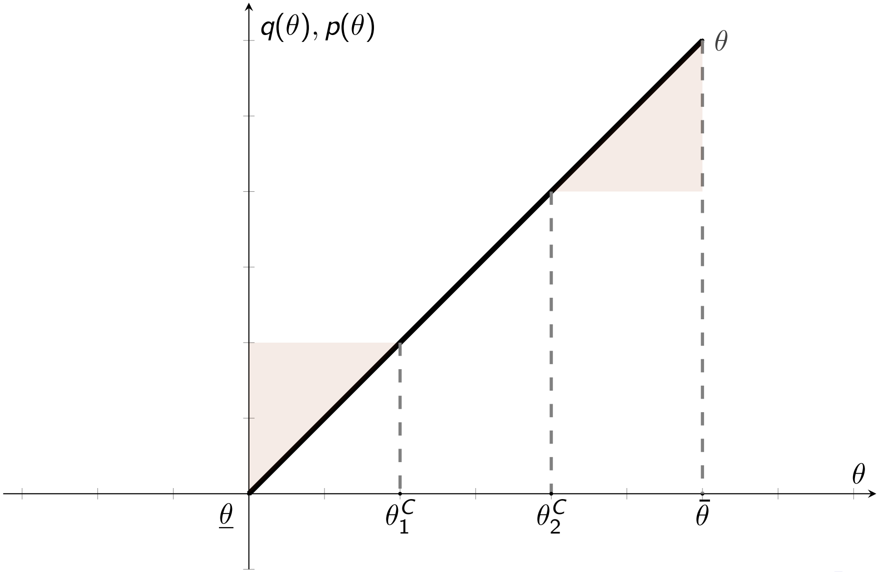
$$\max_{q^S(\theta),q^B(\theta)} \Pi^S = \int_{\underline{\theta}}^{\overline{\theta}} p(\theta)\left[q^S(\theta) - q^B(\theta)\right] g(\theta)\, d\theta$$

subject to the storage constraints.

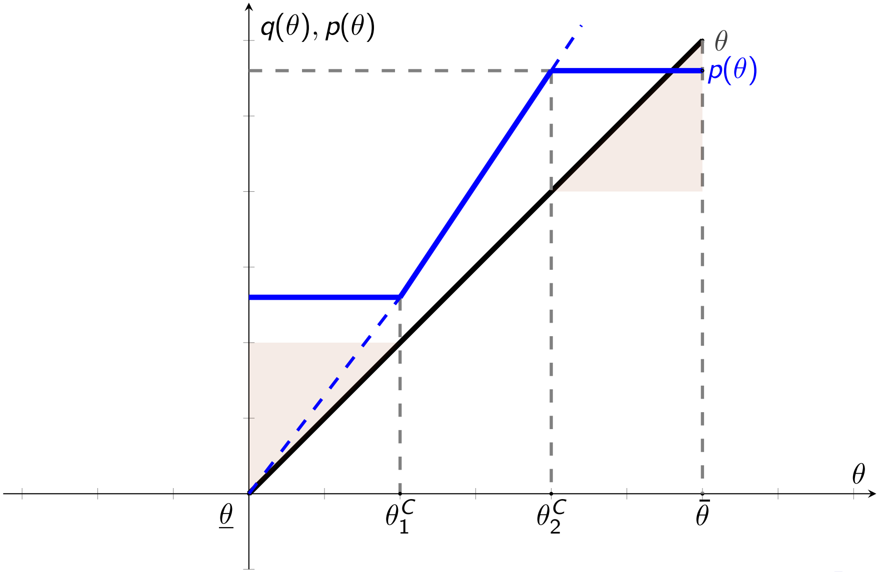
- **Investment stage:**

$$\Pi^S = 0 \rightarrow \int_{\underline{\theta}}^{\overline{\theta}} p(\theta, K)\left[q^S(\theta, K) - q^B(\theta, K)\right] g(\theta)\, d\theta - C(K) = 0$$

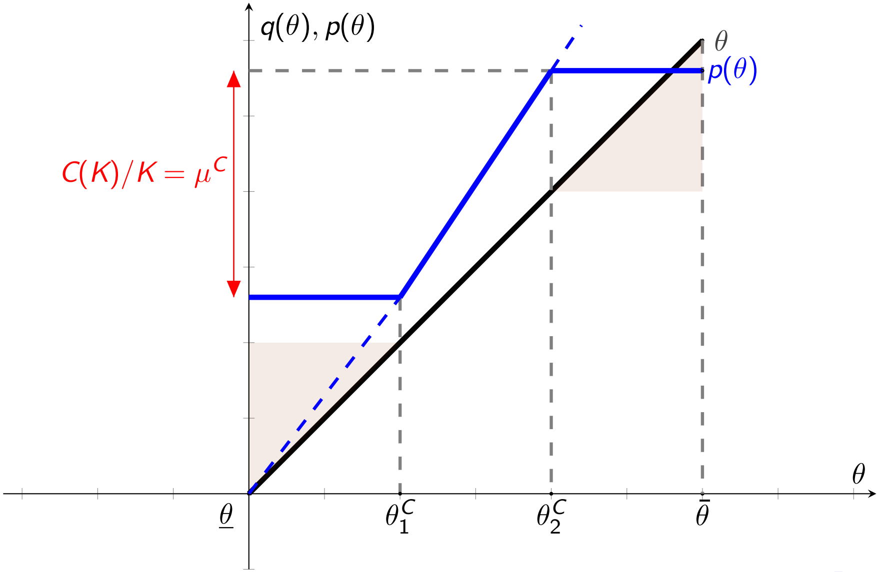
Competitive storage



Competitive storage



Competitive storage

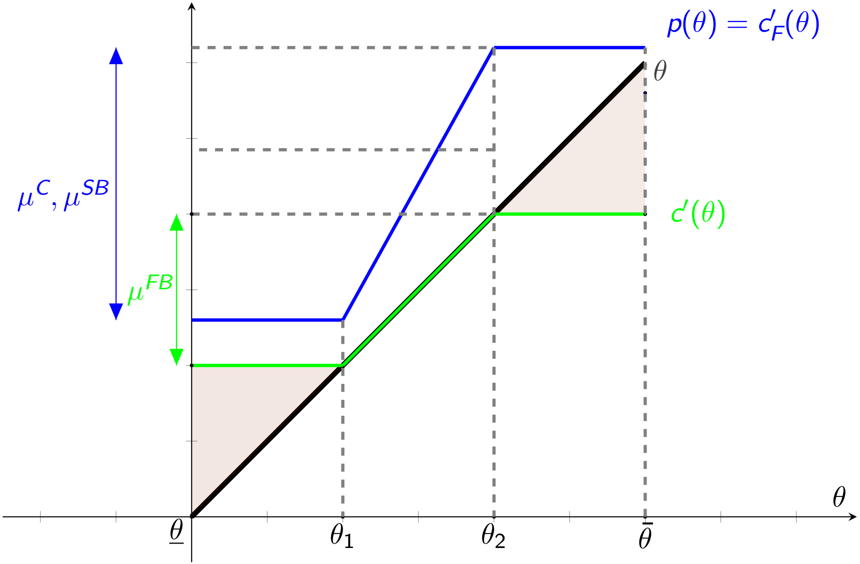


Competitive storage

- **Optimal storage management:**
  - Storage operators exploit arbitrage opportunities.
  - **Prices equalized** within storage and releasing regions.
  - For given K, the price equals the marginal cost of the fringe.
    - Just as in the SB!
- **Equilibrium investment in storage:**
  - Marginal value of storage capacity equals **price differential** that an extra unit of capacity allows to arbitrage.
  - Market power in the product market amplifies arbitrage profits.

Competitive

First Best vs. Second Best vs. Competitive



First Best vs. Second Best vs. Competitive

- Under competitive storage with free-entry in the market, there is **over-investment** and **over-utilization** of storage $\Rightarrow K^{C} > K^{SB} > K^{FB}$.
  - Price differential higher than marginal cost savings.

$$\underbrace{\frac{\theta_2 - \theta_1}{1-\alpha^2}}_{\mu^{C}=\mu^{SB}} > \underbrace{\theta_2 - \theta_1}_{\mu^{FB}}$$

  - Convex capacity costs $\rightarrow$ Infra-marginal profits $\rightarrow C(K)/K < C'(K)$

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  - Convex capacity costs $\rightarrow$ Infra-marginal profits $\rightarrow C(K)/K < C'(K)$

- $K^{SB} > K^{FB} \rightarrow$ Since market power creates production distortions, storage allows for greater cost savings $\rightarrow$ storage is more valuable.

Outline

Storage monopolist: production stage

- **Nash equilibrium** (open-loop strategies).
- **Dominant firm** chooses q_D to maximize "intertemporal" profits:

$$\max_{q^D(\theta)} \pi = \int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{\theta - q^D(\theta) - q^S(\theta) - q^B(\theta)}{1-\alpha} q^D(\theta) - \frac{\left[q^D(\theta)\right]^2}{2\alpha} \right) dG(\theta)$$

- **Storage monopolist** Operate storage so as to max. arbitrage profits (internalizing price effect):

$$\max_{q^S(\theta), q^B(\theta)} \Pi^S = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta - q^D(\theta) - q^S(\theta) + q^B(\theta)}{1-\alpha} \left[q^S(\theta) - q^B(\theta)\right] dG(\theta)$$

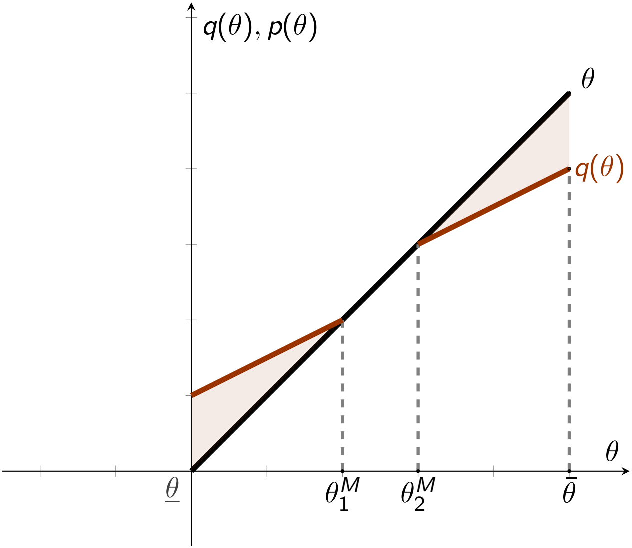
subject to the storage constraints.

Storage monopolist: investment stage

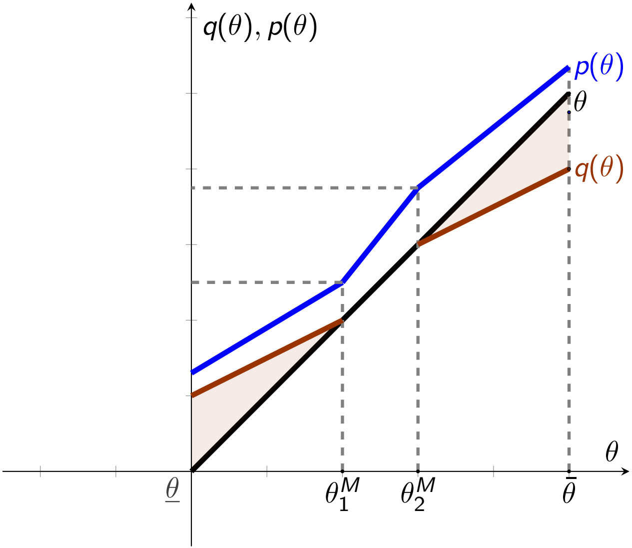
- **Storage monopolist**, given equilibrium production decisions $q^D(\theta, K)$, $q^B(\theta, K)$ and $q^S(\theta, K)$, chooses K to maximize:

$$\max_K \left\{ \int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta - q^D(\theta) - q^S(\theta, K) + q^B(\theta, K)}{1-\alpha} \left[q^S(\theta, K) - q^B(\theta, K) \right] dG(\theta) - C(K) \right\}$$

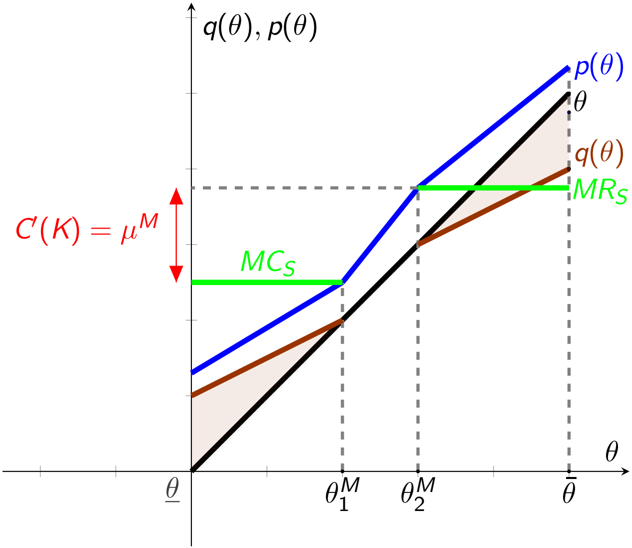
Storage monopolist



Storage monopolist



Storage monopolist

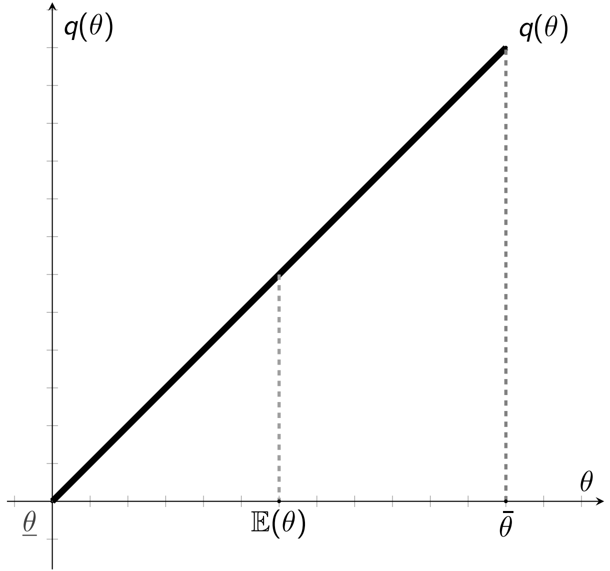


Storage monopolist

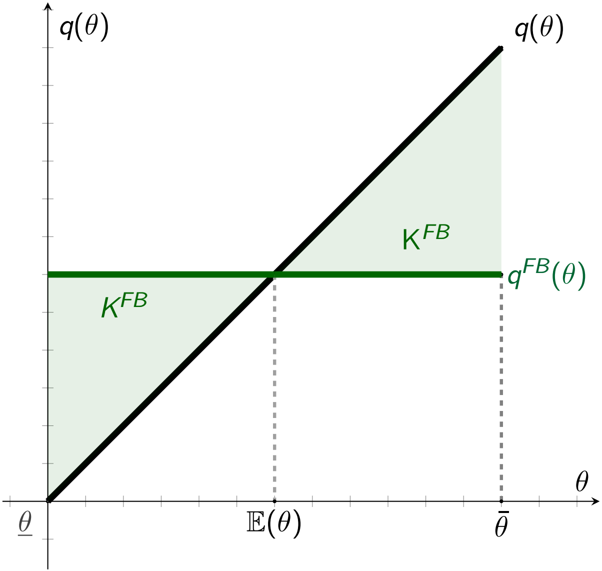
- **Optimal storage management:**
  - Production smoothing $\Rightarrow$ Storage monopolist avoids a strong price increase (decrease) when it buys (sells).
    - Storage monopolist equalizes marginal revenues (costs) when selling (buying).
    - No price-equalization.
- **Optimal investment in storage:**
  - Marginal value of storage capacity equals the **difference between MR and MC** that an extra unit of capacity allows to arbitrage.

▸ Monopolist

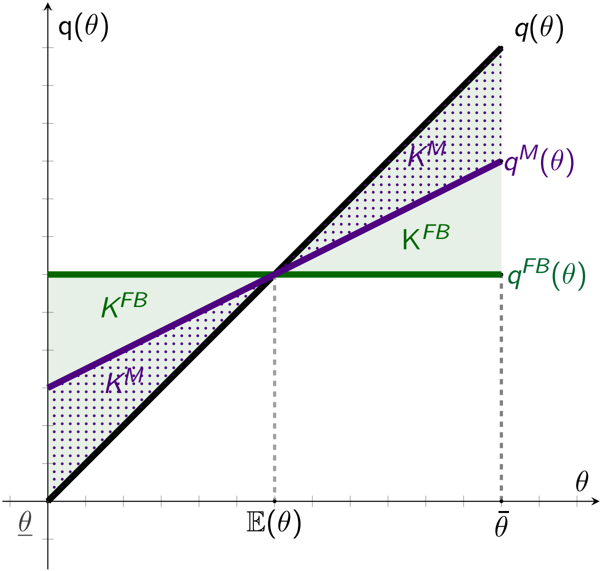
Infra-utilization of storage capacity



Infra-utilization of storage capacity



Infra-utilization of storage capacity



First Best & Second Best vs. Storage monopolist

- **Second Best:** The storage monopolist under-invests $\rightarrow K^M < K^{SB}$.
    1. Market power in storage $\Rightarrow$ lower storage utilization.

First Best & Second Best vs. Storage monopolist

- **Second Best:** The storage monopolist under-invests $\rightarrow K^M < K^{SB}$.
  1. Market power in storage $\Rightarrow$ lower storage utilization.
- **First Best:** Storage monopolist under-invests iff $\alpha < \hat{\alpha}$, with $\hat{\alpha} \in (0,1) \rightarrow K^M \geq K^{FB}$.
  1. Market power in storage $\Rightarrow$ lower storage utilization.
  2. Market power in production $\Rightarrow$ arbitrage profits higher than at the FB.

Outline

Vertically Integrated Firm

- Dominant firm is **vertically integrated** with storage monopolist.

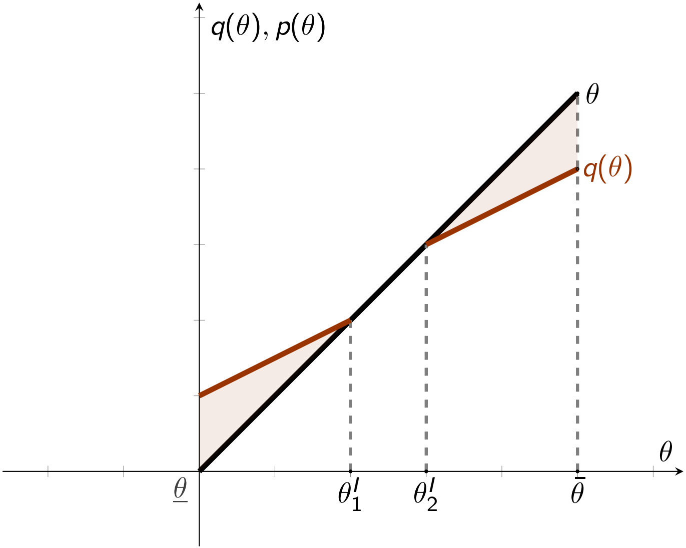
$$\max_{p(\theta), q_B(\theta), q_S(\theta)} \pi_S = \int_{\underline{\theta}}^{\bar{\theta}} \left[p(\theta) D(p;\theta) - \frac{[D(p;\theta) - q_S(\theta) + q_B(\theta)]^2}{2\alpha} \right] g(\theta)\, d\theta,$$

subject to the storage constraints.

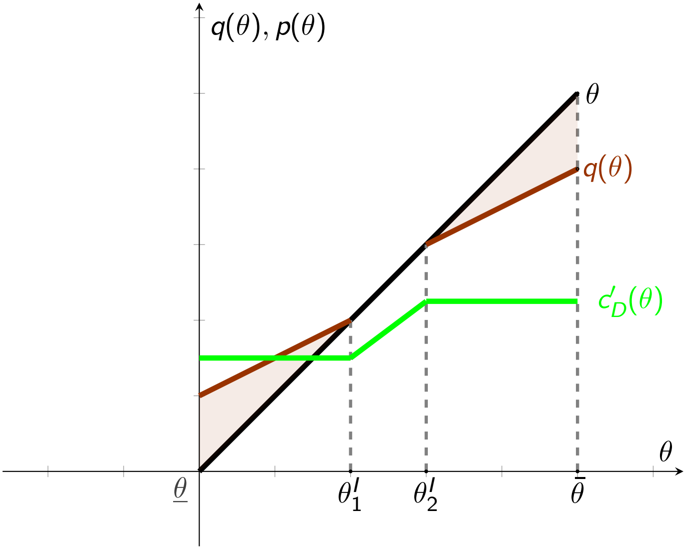
- Higher residual demand (firm controls storage) $\rightarrow D(p,\theta) = \theta - (1-\alpha)p(\theta)$.

- Storage facilities help the dominant producer smooth its production over time.

Vertically integrated firm



Vertically integrated firm



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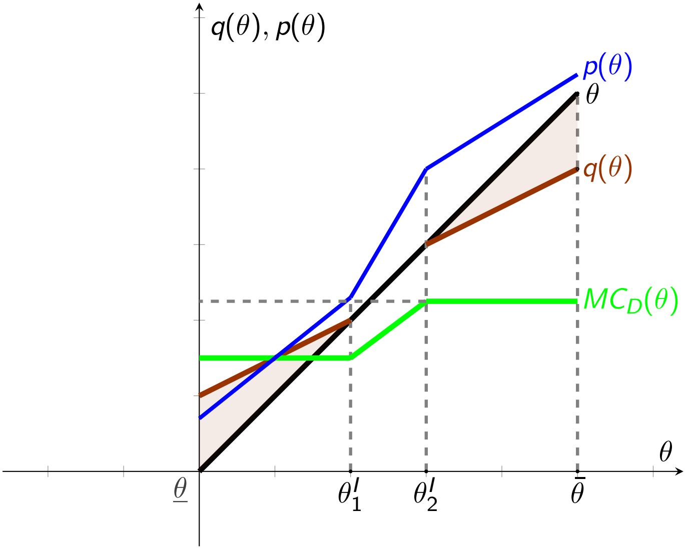

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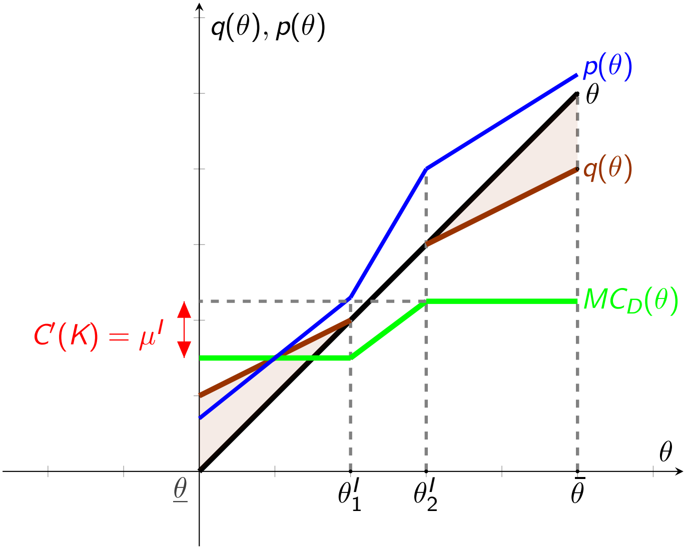

Vertically integrated firm

- **Optimal storage management:**
  - Vertically integrated firm uses storage to **smooth own production**.
  - Under-utilization of given storage capacity with respect to FB.
- **Optimal investment in storage:**
  - Marginal value of storage capacity equals **own marginal cost savings**.
  - Such cost savings are smaller the bigger the firm $\rightarrow$ Storage nvestment decreases in α.

▸ Integrated

First Best vs. Vertically integrated firm

- In a market with a vertically integrated dominant firm, there is **under-investment** in storage, $K^I < K^{FB} < K^{SB}$.
- In contrast to previous cases, K^I is decreasing in α.
  - Efficiency gains from higher α dominate larger arbitrage opportunities
- The under-investment problem is aggravated with respect to the case of an independent monopolist, $K^I < K^M$

Outline

Consumer's surplus

- Consumer's surplus only depends on price profile (demand-weighted average price):
$$CS = v\theta - \int_{\underline{\theta}}^{\bar{\theta}} p(\theta)\theta g(\theta) d\theta.$$
- Market power in production increases all prices, making the price curve steeper.
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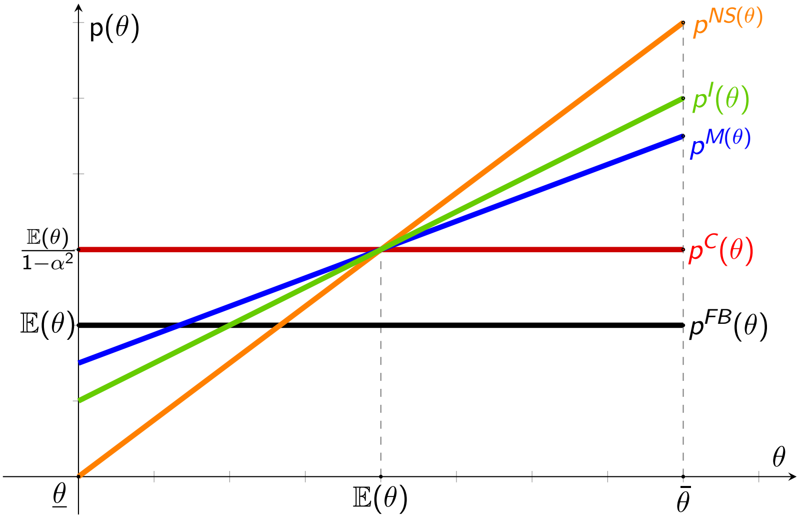
- The ranking of **consumer surplus** across market structures is

$$CS^{FB} > CS^{C} \geq CS^{SB} > CS^{M} > CS^{I} > CS^{NS}.$$

Price profile: no capacity restrictions



Total welfare

- Total welfare is just a function of the **total costs** of production.
- Market power creates static & dynamic productive inefficiencies:
  - Generation (static) $\Rightarrow$ Distorted market shares.
  - Storage (dynamic) $\Rightarrow$ Lower storage usage, production not flatenned.
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Conclusions

- The market does not provide adequate investment incentives in storage capacity.
  - Market power in generation leads to over-investment.
  - Market power in storage to under-investment.
- Vertical integration between storage and generation yields the most inefficient outcome.
  - Texas regulator: utilities are not permitted to use storage.
- Storage reduces the ability to exercise market power in generation, conditional on being independently owned.
- Storage capacity auctions.
  - Solve investment problem, although inefficient storage operation.

Ways forward

- **Empirical simulation** for the Spanish market:
  - Quantify welfare distortions of different market outcomes.
- Introduce other cases.
  - Load-owned storage.
  - Renewable + Storage
- Introduce stochastic demand and/or production.

First Best (cont)

Optimal storage management:

For given K, storage decisions are

$$q_B^{FB}(\theta) = \max\left\{\theta_1^{FB} - \theta, 0\right\} \text{ and } q_S^{FB}(\theta) = \max\left\{\theta - \theta_2^{FB}, 0\right\}$$

where

$$\theta_1^{FB} = E[\theta] - \frac{\mu}{2} \leq \theta_2^{FB} = E[\theta] + \frac{\mu}{2},$$

and $\mu = \mu^{FB}(K)$ is the unique solution to

$$\int_{\underline{\theta}}^{\theta_1^{FB}(\mu)} \left[\theta_1^{FB}(\mu) - \theta\right] g(\theta) d\theta = K.$$

Optimal investment in storage:

$$\frac{\partial \mathcal{W}}{\partial K} = 0 \Rightarrow \mu(K^{FB}) = C'\left(K^{FB}\right) \Rightarrow \theta_2^{FB} - \theta_1^{FB} = C'\left(K^{FB}\right)$$

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Second Best (cont)

Optimal storage management:

For given K, storage decisions are

$$q_B^{SB}(\theta) = \max\left\{\theta_1^{SB} - \theta, 0\right\} \text{ and } q_S^{SB}(\theta) = \max\left\{\theta - \theta_2^{SB}, 0\right\}$$

where

$$\theta_1^{SB} = E[\theta] - \frac{\mu\left(1-\alpha^2\right)}{2} \leq \theta_2^{SB} = E[\theta] + \frac{\mu\left(1-\alpha^2\right)}{2},$$

and $\mu = \mu^{SB}(K)$ is the unique solution to

$$\int_{\underline{\theta}}^{\theta_1^{SB}(\mu)} \left[\theta_1^{SB}(\mu) - \theta\right] g(\theta) d\theta = K.$$

Optimal investment in storage: Back

$$\frac{\partial \mathcal{W}}{\partial K} = 0 \Rightarrow \mu\left(K^{SB}\right) = C'\left(K^{SB}\right) \Rightarrow \frac{\theta_2^{SB} - \theta_1^{SB}}{1-\alpha^2} = C'\left(K^{SB}\right)$$

Competitive storage (cont)

Optimal storage management:

For given K, the equilibrium storage decisions are

$$q_B^C(\theta) = \max\left\{\theta_1^C - \theta, 0\right\} \text{ and } q_S^C(\theta) = \max\left\{\theta - \theta_2^C, 0\right\}$$

where

$$\theta_1^C = E[\theta] - \frac{\mu\left(1-\alpha^2\right)}{2} \leq \theta_2^C = E[\theta] + \frac{\mu\left(1-\alpha^2\right)}{2},$$

with $\mu = \mu^C(K)$ implicitly defined by:

$$\int_{\underline{\theta}}^{\theta_1^C(\mu)} \left[\theta_1^C(\mu) - \theta\right] g(\theta) d\theta = K.$$

Investment in storage:

$$\mu^C(K) = (\theta_2^C - \theta_1^C) / \left(1-\alpha^2\right) = C(K)/K < C'(K).$$

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Storage Monopolist (cont)

Optimal storage management:

For given K, the equilibrium storage decisions are

$$q_B^M(\theta) = \max\left\{(\theta_1 - \theta)/(2+\alpha), 0\right\} \text{ and } q_S^M(\theta) = \max\left\{(\theta - \theta_2)/(2+\alpha), 0\right\},$$

where

$$\theta_1^M = E[\theta] - \mu(1-\alpha^2)/2 \leq \theta_2^M = E[\theta] + \mu(1-\alpha^2)/(2+\alpha),$$

with $\mu = \mu^M(K)$ is the unique solution to $\int_{\underline{\theta}}^{\theta_1^M(\mu)} \frac{\theta_1^M(\mu)-\theta}{2} g(\theta)d\theta = K$

Optimal investment in storage:

$$C'(K) = \mu^M(K) = (\theta_2^M - \theta_1^M)/\left(1-\alpha^2\right)$$

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Vertically Integrated Firm (cont)

Optimal storage management:

For given K, the equilibrium storage decisions are

$$q_B^I(\theta) = \max\left\{\left(\theta_1^I - \theta\right)/2, 0\right\} \text{ and } q_S^I(\theta) = \max\left\{\left(\theta - \theta_2^I\right)/2, 0\right\},$$

where

$$\theta_1^I = E[\theta] - \mu(1+\alpha)/2 \leq \theta_2^I = E[\theta] + \mu(1+\alpha)/2,$$

with $\mu = \mu^I(K)$ is the unique solution to

$$\int_{\underline{\theta}}^{\theta_1^I(\mu)} \frac{\theta_1^I(\mu) - \theta}{2} g(\theta)d\theta = K.$$

Optimal investment in storage:

$$C'(K) = \mu^I(K) = (\theta_2^I - \theta_1^I)/(1+\alpha).$$

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